MEASURING THE VELOCITY IN A BOUNDARY LAYER WITH A TOTAL-HEAD PITOT TUBE

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UDC 532.574

Results are shown of an experimental study concerning the effects of viscosity, of the velocity gradient, and of the wall proximity on the readings of a total-head Pitot tube during the measurement of the velocity distribution in the viscous sublayer region of a turbulent boundary layer.

The use of total-head Pitot tubes for measuring the velocity in a boundary layer directly at the wall involves serious difficulties stemming from the lack of necessary data on the performance of such tubes under these conditions. As is well known, errors may occur in the measurements due to:

a) the effect of velocity gradients in the boundary layer and the effect of the wall proximity on the tube readings,

b) the effect of viscosity, i.e., the inapplicability of the ideal-fluid theory in the interpretation of total-head measurements made directly at the wall, where the Reynolds number is low.

The deviation of the measured velocity distribution in a boundary layer from the true distribution may be related either to the error in determining the magnitude of the velocity or to the error in determining the distance from the tube to the wall. The second alternative of defining the error is usually preferred, a correction being made here for the displacement of the effective center from the geometrical axis of the tube.

The error in velocity measurements due to velocity gradients in the stream has been evaluated in several studies theoretically (e.g., [1, 2]) and experimentally ([3, 4, 5]), but there is no consensus on this subject. Still not enough is known about the effect of the wall proximity on the readings of a total-head Pitot tube; such an effect being dependent on the tube geometry as well as on the flow mode in the boundary layer [5].

Using the total-head Pitot tubes with a small diameter for measuring the velocity in a thin viscous sublayer of turbulent boundary layers makes it necessary to take into account the effect of viscosity on the tube readings. In this case the Reynolds number, when calculated on the basis of local velocity and characteristic tube dimension, is very low and the Bernoulli equation, when derived by integrating the equations of motion simplified by neglecting the viscous forces, does not apply when the velocity is to be found from a pressure measurement with a total-head Pitot tube. The effect of viscous forces, which become comparable to the inertia forces, is to increase the total head relative to the head calculated according to the Bernoulli equation, because now the pressure coefficient $Cp = (P_0-P)/(\rho U^2/2)]$ becomes larger than unity. The data in [6] and [7] pertaining to the effect of viscosity on the readings of a total-head Pitot tube are very much at variance. Furthermore, no tests were made in those studies with the Reynolds number $Re_R < 10$, which would be of interest for measurements in a viscous sublayer.

For practical purposes it would be worthwhile to know the total correction δ_0 accounting not only for the effect of viscosity but also for the effects of the velocity gradient and the wall proximity on the readings of a total-head Pitot tube.

N. E. Zhukovskii Central Institute of Aerodynamics, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 22, No. 3, pp. 505-510, March, 1972. Original article submitted February 24, 1971.

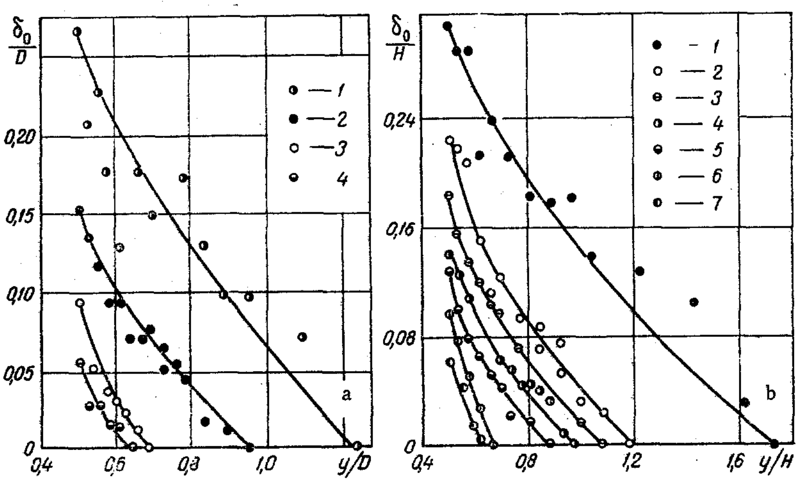

Fig. 1. Graphs representing the total displacement of the effective center in a total-head Pitot tube: (a) with a circular inlet orifice, U_*D/ν = 7.12 (1), 8.85 (2), 13.85 (3), 15.72 (4); (b) with an elliptical inlet orifice, U_*H/ν = 5.11 (1), 6.65 (2), 8.43 (3), 9.72 (4), 10.31 (5), 11.92 (6), 12.9 (7).

We will present here the results of an experimental determination of the total correction and its components, to be used when the velocity distribution in a turbulent boundary layer of an incompressible fluid is measured by means of a total-head Pitot tube with a circular or elliptical inlet orifice.

The procedure in this experiment is based on the fact that the effects of viscosity, velocity gradient, and wall proximity are felt directly at the wall, namely in the region of the viscous sublayer where during a nongradient flow of liquid or gas the velocity distribution would be precisely linear (as many measurements by various optical methods have shown). The total correction δ_0 to be applied to tube readings to account for all these effects will be determined by the difference between the true linear velocity distribution and the one measured with a total-head Pitot tube at a wall.

The slope of the line representing the true velocity distribution at a wall may be determined from a measurement of the shearing stress τ_w, the accuracy of such a measurement affecting the reliability of the measured velocity distribution at that wall.

In our tests τ_w was determined directly by a measurement of the friction force by the weighing method with the aid of a "floating" element. For this, we used a high-sensitivity electromagnetic scale [8] in the 0.5-100 mg range. The relative mean-square error in this determination did not exceed 1.5%.

The magnitude of the correction δ_0 as a function of the distance from a wall is shown in Fig. 1. for different values of the Reynolds number with a circular and with an elliptical total-head Pitot tube. The observed dependence of δ_0 on the Reynolds number, as will be proved subsequently, is due primarily to the effect of viscosity.

The "pure" effect of viscosity on the tube readings was studied in a low-velocity vertical aerodynamic tunnel, especially designed for this purpose, where air was impelled directly from the atmosphere by means of a fan mounted at the outlet of the active tunnel segment (Fig. 2). The total-head Pitot tube for this test was placed along the tunnel axis near the profiled air impeller and beyond the boundary layer building up at its walls. It could be assumed, thus, that $P + (\rho U^2/2) = P_{atm}$ and the pressure coefficient $C_P = 1 + [(P_0 - P_{atm})/(\rho U^2/2)]$. For an accurate measurement of the pressure drop $\Delta P = P_0 - P_{atm}$, we had developed a differential-type precision U-tube manometer with alcohol as the operating medium [9], with the alcohol level and the instrument readings tracked automatically by means of photodiodes, optical lenses, a relay system, and an electromagnetic plotter. The mean-squared error of the manometer did not exceed σ = 0.01 mm H_2O within the ΔP = 0-2 mm H_2O range. A reduction of the random error was achieved by performing a large number of measurements (up to twenty) under the same conditions.

The values of C_P are shown in Fig. 2 for total-head Pitot tubes with circular and with elliptical inlet orifices. When $Re_R < 30$, evidently, C_P increases sharply and for a circular tube $C_P \approx 2.5$ already at $Re_R = 5$. For a circular tube, the values of C_P obtained in our tests are more than twice as high as

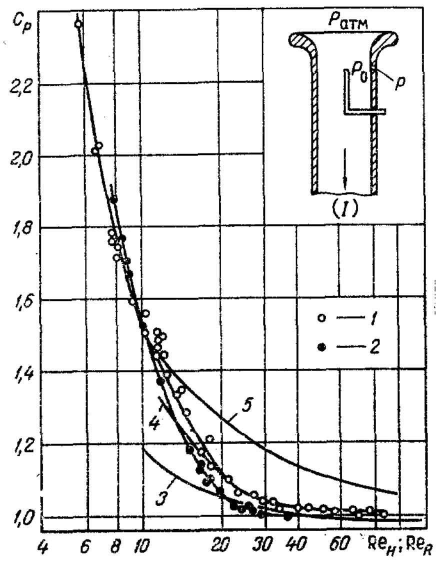

Fig. 2

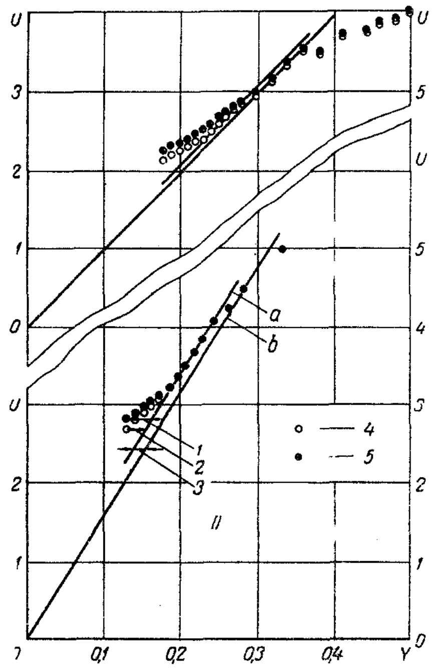

Fig. 3

Fig. 2. Coefficient C_P as a function of the Reynolds number Re_R or Re_H: (a) Schematic diagram of the aerodynamic test apparatus: data of this study with a circular Pitot tube d/D = 0.58 (1), data of this study with an elliptical Pitot tube h/H = 0.46 (2), data of [7] for a circular Pitot tube d/D = 0.74 (3), data of [6] for a circular Pitot tube d/D = 0.6 (4), calculations according to F. Homann [1] (5).

Fig. 3. Typical velocity distribution at a wall, measured with a circular (I) and with an elliptical (II) total-head Pitot tube: δ_0 (1), δ_2 (2), δ_1 (3), with a correction for the effect of viscosity (4), without a correction for the effect of viscosity (5). Velocity U (m/sec), distance Y (mm).

those obtained in [7], but they agree with those in [6] within the Re_R = 17–40 range. The discrepancy with [7] could possibly arise from the fact that the d/D ratio of the tubes there was much higher than in our tests (Fig. 2).

For tubes with circular inlet orifices and d/D ≈ 0.6 we recommend the following empirical formula:

$$C_P = \frac{Re_R}{1{,}1\ Re_R - 4{,}26} \quad \text{at} \quad 5 < Re_R < 35. \tag{1}$$

For elliptical tubes, the highest Reynolds number Re_H at which the effect of viscosity remains still significant is somewhat lower than Re_R for circular tubes. The height of an elliptical inlet orifice, on the basis of which the Reynolds number is calculated for given tests, does not appear to be the characteristic dimension here and, consequently, the $C_P = f(Re_H)$ curve in Fig. 2 will be valid only for a specific tube whose shape and dimensions may be recommended for practical measurements in a boundary layer.

The correction for the velocity gradient and for the wall proximity will be based on the following considerations. According to the velocity distribution in a viscous sublayer shown in Fig. 3, after the correction for viscosity has been added, the test values for points already sufficiently remote from the wall fit on a straight line *a* parallel to line b, the latter passing through the origin of coordinates after having been plotted on the basis of shearing stress τ_W measurements at the wall by the weighing method. The distance between these two lines may be said to determine the displacement of the effective tube center (δ_1) owing to the effect of the velocity gradient on the total-head readings. The deviation of test points from line *a* directly at the wall, where a constant velocity gradient prevails, will be attributed to the effect of wall proximity.

For a linear velocity distribution at the wall, according to this determination, the value of δ_1/D was almost independent of the velocity gradient and approximately equal to 0.03 for the circular inlet orifice ($D = 0.354$ mm and $d/D = 0.58$). Thus, the value of δ_1/D obtained in our tests was much smaller than $\delta_1/D = 0.15$ suggested in [4]. There is reason to believe that the value of δ_1/D will depend on the tube diameter.

For the tube with an elliptical inlet orifice ($H = 0.26$ mm, $L = 1.1$ mm, and $h/H = 0.46$) the value of δ_1/H was 0.06 and almost independent of U_*H/ν.

As to the effect of wall proximity, in the case of an elliptical tube (with a nearly flat shape) it is somewhat stronger than in the case of tubes with circular inlet orifices and it extends up to $Y/H = 0.8$.

NOTATION

C_P is the dimensionless pressure coefficient;
$Re_R = UR/\nu$ is the Reynolds number based on the outside radius of a circular total-head Pitot tube;
$Re_H = UH/2\nu$ is the Reynolds number based on half the outside height of the elliptical inlet orifice of a total-head Pitot tube;
P_0 is the pressure, measured with a total-head Pitot tube;
P is the static pressure at the section where a total-head Pitot tube is placed;
P_{atm} is the atmospheric pressure;
U is the velocity of unperturbed stream;
$U_* = \sqrt{\tau_W/\rho}$ is the rate of friction;
τ_W is the shearing stress at the wall;
ρ is the air density;
ν is the kinematic viscosity of air;
D, d are the outside and inside diameters of total-head Pitot tube respectively;
H, h are the maximum outside and inside heights of the elliptical inlet orifice of a total-head Pitot tube respectively;
L is the width of the elliptical inlet orifice of a total-head Pitot tube;
Y is the distance from the wall to the geometrical center of a total-head Pitot tube;
δ_1 is the displacement of the effective tube center due to the effect of the velocity gradient;
δ_2 is the displacement of the effective tube center due to the effect of the wall proximity;
δ_0 is the displacement of the effective tube center due to the combined effect of the velocity gradient, the wall proximity, and the fluid viscosity.

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